Cambridge Assessment International Education

Teacher Guide

Incorporating language learning support

Cambridge IGCSE®

Additional Mathematics 0606

Cambridge O Level

Additional Mathematics 4037

For examination from 2020

Version 1.0

Cambridge Pathway

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Contents

Introduction

The purpose of the teacher guide

This teacher guide is designed to help you to organise and plan your teaching for Cambridge IGCSE or O Level Additional Mathematics. Note that throughout the document is refers to the IGCSE course but the information applies equally to the O Level course. The guide includes advice and guidance on teaching strategies and how to prepare your learners for the final assessment, with a particular focus on preparing for problem solving and multi-technique assessment questions.

As an international awarding body, many of our candidates are either multi-lingual or possess English as a second language, which presents them with great opportunities but also with potential barriers. Learners cannot develop academic knowledge and skills without access to the language in which they are discussed, constructed and evaluated. Therefore in this guide, we have included some prompts and tips on how to incorporate the development of language skills within the general teaching of this subject.

The information relating to language support is indicated using the icon shown here.

Where language levels are already highly developed amongst learners, this advice will not be applicable. However, it is often the case that those learners with lower ability skills in general and those with lower ability language skills tend to share similar misunderstandings.

Getting started

School Support Hub

You should make sure at an early stage that you have access to the School Support Hub, www.cambridgeinternational.org/support. You can obtain a login from your Examinations Officer.

The School Support Hub provides a wide range of resources to help you, including:

- syllabuses
- past examination papers and specimen papers
- mark schemes
- Principal Examiner Reports for Teachers (after the first sitting in 2020)
- Example Candidate Response booklets (after the first sitting in 2020)
- Schemes of Work
- Specimen Paper Answers (before the first sitting in 2020)
- community resources and discussion forum.

All of these forms of teacher support are invaluable in helping you and, in turn your learners, understand exactly what Cambridge expects of candidates in examinations, and will help you to prepare your learners appropriately.

Syllabus

When planning your course, your starting point should be the syllabus. This contains information not only on the curriculum content but also the overall aims and assessment objectives. It gives details of the examination papers, the required equipment and additional information. It is important that you become thoroughly familiar with all parts of the syllabus document.

Scheme of work

You will then need to devise a scheme of work. You need to think about how you will organise the time that you have available to help learners to understand and learn all of the facts and concepts required by the syllabus, and to develop the necessary skills. Cambridge provides a scheme of work that you could use as a starting point but you will undoubtedly want to produce your own eventually. Your scheme of work will help you to determine what resources you will require to deliver the course and this will help you to build up teaching, learning and reference resources such as text books and worksheets.

1 Planning the course

This section looks at how you can plan your course to ensure that you can cover the whole syllabus within the time that you have available. There are three main types of planning: long-term, medium-term (developing a scheme of work) and short-term (planning for individual lessons). This section also includes ideas and support for incorporating language into the lesson to help learners become more fluent and accurate users of English.

Language is an important part of communication and you should make sure that your learners complete given tasks in English. The language focus is not an additional element to be added to the course but should be seen as the medium through which the topic content is taught. The promotion of critical thinking skills and collaborative work is considered to be very important in acquiring language and improving fluency. Personalisation of the topic, where possible, is also known to increase motivation and self-confidence as well as interest in the topic. Section 3.3 provides some ideas and activities for incorporating language into the lesson without increasing your workload.

1.1 Key factors to consider when planning your course

These factors will need to be considered before starting the planning of your course:

- the amount of teaching time available each week for the duration of the course
- the availability of resources such as graphical calculators or graphing software
- your learners' prior knowledge
- your learners' level of English
- whether your group is mono- or multi-lingual
- whether your teaching groups will be mixed ability or will be streamed by ability
- the number of lessons you will need to cover the syllabus (the recommended time for an IGCSE course is 130 hours of teaching time)
- the school calendar; holidays, examinations, etc.

1.2 Long-term planning

A long-term plan will provide the overall structure of your course. It will include the order in which topics will be taught, the approximate length of time to be spent on each and the factors listed in section 1.1 above. It will need to take into account the number and nature of the groups following the course and whether they should all follow the same path through the course. There may, for example, be issues with the use of computer equipment if two groups are studying a topic requiring a large amount of graphical work at the same time. In this case it would be better if the plan was organised so that groups could study such a topic at different times.

Topics should also, ideally, be arranged so that they fit into the school's sessions, so that a topic is not split because of a school holiday or an examination session. In a two-year course, the second year will probably have fewer weeks because of the timing of the Cambridge examinations.

It is important to note that you do **not** need to teach the subject content in the order in which it is printed in the syllabus. It is likely that you will want to order your teaching to suit your learners' particular needs and preferences. This may be done in a number of ways.

You could:

- Start your course with Section 2 (Quadratic functions) as your learners will be familiar with many of the initial concepts and there is plenty of opportunity for introducing the use of technology to enable learners to visualise results, or perhaps you might start with this section because solving quadratic equations is an essential skill for the course and underpins many other sections of the syllabus.
- Start with topics which are conceptually easier, saving the more difficult topics for the second part of the course.
- Use the suggested teaching order given in the 'Schemes of Work' provided on the School Support Hub (www.cambridgeinternational.org/support).
- Follow your own and learners' interests to begin with, starting with topics that you are all enthusiastic about.

Long-term planning will also consider what you would like the learners to able to communicate, either in spoken or written form, at the end of the course. This will help with identifying what language could be included in medium-term and short-term plans.

A long-term plan should also consider how problem-solving skills will be developed and which topics will contribute largely to the development of these skills. This is particularly important because the ability to apply techniques from different parts of the syllabus to solve problems is a fundamental part of the Assessment Objectives for the course.

A long-term plan is a flexible, working document. As the course progresses, you can adapt it as required. When you have worked through it once or twice you will have a much better idea of the best way for you to work through the syllabus.

1.3 Medium-term planning

Medium-term planning is the most important of the three types. It defines in moderate detail what will be taught and when. It also describes how language skills, practical work and other activities are to be incorporated into the course. Medium-term plans are often called 'Schemes of Work' and can be shared at the whole school level to inform other mathematics teachers of your plans.

Cambridge has produced one example of a scheme of work for this course, which is available from the School Support Hub (www.cambridgeinternational.org/support).

The Cambridge scheme of work is useful as:

- an example of **one way** of moving through the course
- a good source of possible activities mapped to each part of the syllabus
- a good source of exercises and resources.

Generally, we recommend that you only use this as a reference to help you create your *own* scheme of work because, it:

- represents only **one** possible approach and so is not necessarily the best for your learners
- does not take into account the ethos, approach and/or facilities of your centre
- is arranged in a way that might not fit in with your long-term plan
- has no statement of the amount of time required for each element.

When adapting an existing scheme of work:

- add timings for each section
- you could include a note about the sort of output you would like your learners to produce in terms of language, e.g. oral, written, group/pair work, discussions, etc.
- always check the URLs before using them with your learners; web addresses can change, and you also need to know that what you are accessing is appropriate for your learners.

A medium-term plan is best developed with contributions from all of the teachers who will be using it. If they have had an input they will feel an 'ownership' of the plan and will be more likely to adhere to it.

A medium-term plan should be flexible and updated when necessary. It should be amended if it is found not to be working as planned. It should be reviewed at the end of each school year to assess how well it has worked and to decide if any improvements need be made.

1.4 Short-term planning

Short-term planning involves planning for a single lesson or perhaps a small group of lessons.

It should include the:

- content and the language of the lesson
- activities that will take place
- progress that is expected of the learners during the lesson.

Short-term planning is something that is done by an individual teacher, taking into account their own strengths and the needs of the learners they will be teaching. Teachers new to the subject may need guidance but the plan should still be their own.

This process is covered in more detail in the next section, *2 Planning lessons*.

2 Planning lessons

2.1 Lesson plans and templates

A lesson plan is written by the teacher and should include details of how the lesson is intended to proceed. It should take account of:

- what is to be taught (learning objectives)
- what is to be achieved by the learners (lesson objectives, content and language)
- what the learners already know (previous learning and relevant knowledge)
- how learners understanding will be monitored (assessment of learning)
- how learners at different levels of ability are going to access the lesson (differentiation).

It should detail the learning activities that will take place, and have approximate timings for how long each part of the lesson will last. It should also briefly note the language focus for the lesson.

A lesson should ideally have three main parts:

beginning	this should be an activity that engages and motivates the learners, as well as stimulating the background knowledge that the learners can bring to the topic
middle	this should include the main learning and language activities of the lesson
end	this should be an activity/activities in which learners can assess their understanding of what has gone before and feedback on it.

2.2 Constructing a lesson plan

It can be helpful to have a printed template to use in lesson planning. You can design your own, or there are many available on the internet or in books. On the following pages you will find an example of a completed lesson plan which includes helpful guidance.

A blank template of the example below is available in the Appendix for you to copy.

Lesson:	**School:**	
Date:	**Teacher name:**	
Class:	**Number present:**	**Number absent:**
Learning objectives to which this lesson is contributing	*This will be based on something written in your medium-term plan. It will state which part of the syllabus the lesson is going to address. It could, for example, be a syllabus statement.*	
Lesson objectives	*These objectives are what you intend the learners to fully understand and be able to do by the end of the lesson. It should be a list of outcomes that the lesson intends to target. Do not include too many outcomes – any target should be realistic and some learning objectives will take more than one lesson to be fully understood. You could include some indication of differentiation here by having some outcomes that are for 'all' learners, some that are for 'most' learners and some that are only for the best learners ('some').*	
Vocabulary, terminology and phrases	*Any notation, key words or terms that are essential to the understanding of the lesson can be listed here for ease of reference.*	
Previous learning	*Write down any prior learning required from other courses or from within this course. Learners' confidence might be increased by knowing what existing skills they will be using in the lesson. They will also be mentally aware of what tools they need in advance of needing them, which can improve understanding. It also is a good opportunity to show learners the inter-connectedness of mathematics.*	

Plan

Planned timings	Planned activities	Resources
Beginning	*A starter activity should be a short introduction to the skills needed for the lesson. It should engage learners with mathematics. It can be a short question and answer session, a simple quiz to review prior learning, or a look at the common errors made in a topic. It could even be a rapid practical demonstration to introduce them to the topic to be covered in the lesson.* *A starter should also stimulate the interest of the learner by providing materials such as visuals for the particular vocabulary needed; or some activity that is personalised to encourage the learners to bring their own background knowledge and interest to the topic. It is vital that starter activities are not laborious; many good ideas and examples are available online.* *Give an estimated time, usually about five to ten minutes.*	*Your plan should also include a list of the resources that will be needed in each lesson. Examples include, course books, internet access, laptops, calculators, and graphing software.*

Plan		
Planned timings	**Planned activities**	**Resources**
Middle	*This is the main part of the lesson. It can build on and extend previous understanding, explore and solve practical problems, develop knowledge and skills, practise previously learned techniques, or any of a number of other alternatives.* *It is important not to include too many activities, but equally important not to spend so much time on one activity that learners become de-motivated. There may be opportunities for investigation, explanation, practice and application of their newly acquired skills. Good lessons will involve learners in all aspects of the lesson as much as possible. Activities should encourage the learners to have confidence in communication through speaking or writing and there should be the opportunity to give feedback.* *Timings should be included for each separate activity.*	
End	*This is an important part of the lesson as brings it to an organised end. It also gives you the chance to assess how well learners understand the material covered during the lesson. As with starter activities, this could be a short written exercise, a quiz, or a question and answer session. This is an excellent opportunity to re-examine key words and terms and possibly start to make a glossary of these to present on the wall.* *This part of the lesson could also be used to link to whatever is going to happen in the next session.* *This part of the lesson should take around five to ten minutes.*	

Additional information		
Differentiation: How do you plan to give more support? How do you plan to challenge more able learners?	**Assessment: How do you plan to check learning?**	**Health and safety check: ICT links**
Outline how will you try to ensure that the lesson is accessible to all of the learners so that all will benefit from the experience. This is especially important with mixed ability groups. There is more on differentiation in section 3.	*Assessment for learning can take place all through the lesson. For example, by observation, short Q & A sessions, listening and asking directed or open questions, quizzes, homework, learners presenting the results of an investigation to the class, or by teacher-marking of classwork or homework. Do this to discover what your learners knew/understood before the lesson and how this has changed after the lesson.*	*If your lesson includes the use of technology such as laptops, or the use of external websites, the equipment should be safe and the external websites should be checked to make sure they are also safe. An assessment of the risks involved should be included with the lesson plan.*

Reflection and evaluation	
Reflection *Were the lesson objectives realistic?* *What did the learners learn today?* *What was the learning atmosphere like?* *Did my planned differentiation work well?* *Were the timings correct?* *What changes did I make from my plan and why?*	**Use the space below to reflect on your lesson. Answer the most relevant questions from the column on the left about your lesson.** *As soon as possible after the lesson you need to think about how well it went. This reflection will be helpful next time you teach the same topic. If the timing was wrong or the activities did not fully occupy the learners this time, you might want to change some parts of the lesson next time.* *It is a good idea to discuss with colleagues how your lesson went, good or bad. They might have valuable advice to offer you and such collaboration can help you to develop your own teaching skills. Sharing your lesson plan with other teachers in your centre will also enable them to learn from your experiences.* *There is no need to re-plan a successful lesson every year, but it is always good to learn from experience and to incorporate improvements next time.*

Summary evaluation

Write down two things that went really well. (Consider both teaching and learning.)

1.

2.

Write down two things that would have improved the lesson. (Consider both teaching and learning.)

1.

2.

What have I learned from this lesson about the class or learners that will improve my next lesson?

3 Classroom practice

The aim of any teacher is to get their learners to gain knowledge and understanding, to develop the skills to be able to apply this knowledge, and to learn to communicate what they know as effectively and accurately as possible in the time available to them on the course.

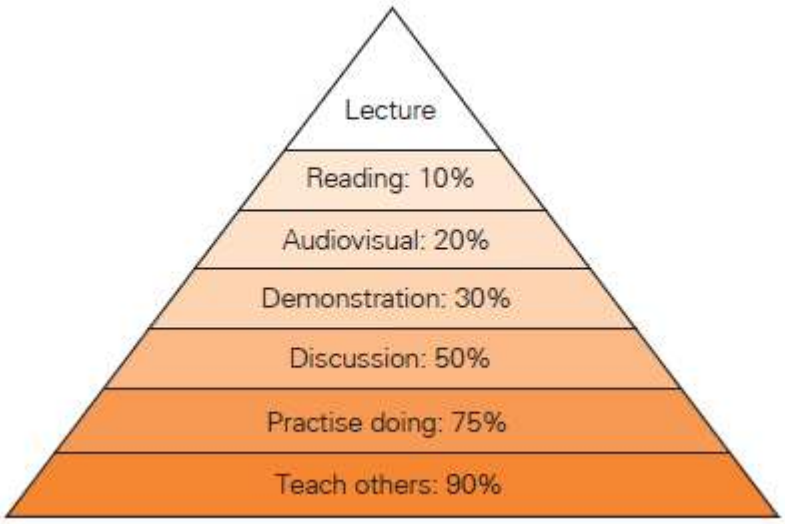
The teaching should take account of the different needs and abilities across the full range of learners represented in the group. Lessons should be interesting and involve the learners as much as possible.

3.1 Active learning

A description or explanation by the teacher is easily forgotten by the learner, even when it is understood. Videos and computer animations of some mathematical techniques can help, but they are still 'passive'. The learner is not involved in 'discovering' the information.

Research has shown that the more a learner is involved in the process of learning, the more they retain. This is also true of language acquisition.

The learning pyramid below shows the percentage of information retained as a result of different forms of delivery, stimulating different learning processes.



From this it can be seen that although audio-visual (videos and computer animations) may be better than a lecture (being told by a teacher) there are methods that are better still. Clearly not everything can be absorbed by discussion and practice, but activities where the learners actually participate work better.

At least some such activities (active learning) should be used alongside problem solving/investigative work in order to maximise learning. There will not be time for everything to be covered in this way but some topics certainly should be. If you give learners guiding questions to answer while listening, this will activate knowledge and language and will allow the learners to feedback the answers or contribute to the group discussion more effectively. This is an example of what is called 'scaffolding'.

There are, of course, many other methods of getting learners involved and plenty of ideas in books and on the internet.

3.2 Differentiating the activities

Differentiation is a way of trying to ensure that learners in your classroom with differing abilities can all access the material you are delivering. There are a number of ways of approaching this problem and, again, they can be found in books and on the internet. They fall into three main categories.

- **Differentiation by outcome** – In this method, an open-ended task is set that can be accessed by all, e.g. 'Investigate the properties of a series whose sum to an infinite number of terms approaches a limit'. Learners will produce different results according to their ability, but all of their 'outputs' will be valid.
- **Differentiation by task** – Learners are set slightly different tasks based on the same objective. This could involve, for example, worksheets that pose questions on the same topic but which require different amounts of understanding.
- **Differentiation by support** – All learners undertake the same task but less able learners are given additional support. Writing frames, where a template is provided for them to record their work, are one way of doing this.

3.3 Integrating content and language in the course

In previous sections we have stressed that the objective of the language element of a lesson is to help the learners gain greater confidence in communicating their knowledge of the subject. Whether you are teaching a class including learners who have English as their second language, who are multi-lingual or who only speak English, the same difficulties of written expression occur within the subject to both lower level ability and second language learners.

Subject teachers are not expected to teach English, however, mathematical language and terminology should be learned at the same time as the subject content, as a fluent part of the content, so that it has greater meaning and offers contextual understanding; mathematical language should not be left to a specified language lesson. The confidence to communicate in accurate and precise language will also be of benefit to learners taking an external examination in English at the end of the course. A number of examination questions require explanations and learners can gain higher marks if the language is used accurately. For example, comments by some examiners on previous candidate scripts have noted the incorrect use and understanding of vocabulary, the lack of ability to write a logical explanation and answers that contain contradictions. It will also benefit learners in the long term, should they continue their studies in the subject at a higher level with a view to their careers.

The teacher's role should, therefore, also be to support the language element of the lesson that underpins the content. This element should enhance learners' communicative skills and their accurate use of the language. A key part of this should be for teachers and learners to notice the language used in different stages of the lesson.

Here are some strategies that you can try in your next lesson:

- record language prompts on the whiteboard
- encourage learners to underline key terms
- use images
- provide writing frames
- enable learners to write collaboratively
- introduce learners to new language before setting a task
- provide sentence stems and model language
- activate prior knowledge of the subject
- create a bank of useful expressions
- repeat explanations and progressively increase the difficulty of explanations

- provide feedback on language and content
- highlight examples of good language use from learners.

To help learners with their use of language it may also be helpful to consider the following questions when writing a lesson plan for a subject area:

- What is the topic and what does it cover? (content)
- Is there something in the topic you can make personal to the learners? For example, is there something you can relate to their particular culture to stimulate interest and prior knowledge. (context, personalisation)
- What language will your learners need to produce during the lesson and later in the exam? (English)
- What is the language focused on? For example, it may be to explain a result, justify a decision, describe a result, or interpret given information. You can also think about the relevant vocabulary and terms they could practise to help with precision.

3.3.1 Some ways to integrate content and language

Language is the medium through which the content is delivered. However, your principal aim is not to teach the language but rather to provide language support, and use it in interesting ways. Mathematics has a symbolic language of its own, of course, and this should also be considered along with learners' understanding of written English. The following suggestions should help you think about what might support learners with the language during the lesson:

- Use of visuals and charts for building vocabulary and understanding meaning – this has been shown to stimulate interest and the learners' prior knowledge.
- Use gap fill and word definition to discover meaning – this helps with retaining the language to a greater extent than when learners are simply given the answers.
- Use checking questions to ensure understanding – asking learners if they could tell the group or their partner what they have to do is an important part of communication and retention.
- Pair and group work is important – learners learn from each other and it has been shown that teenagers prefer to work in groups rather than on their own. Learners need a safe place to practise the language before expressing it individually.
- Oral interaction between learners in English about the content is beneficial as is cooperative work. The more the learners speak, the greater the development of accuracy and confidence in using the language.
- Repeat vocabulary, grammar and useful phrases commonly used in the topic. For example, practise using words to explain an idea, to justify or develop an argument.
- When learners are encouraged to notice the language, they are more likely to use it at a later date.
- The use of context, where appropriate, is important for learners to understand meaning.

3.3.2 Possible activities

(1) Activate prior knowledge by using simple visual clues to vocabulary, match pictures to words.
(2) Personalisation – start the lesson by making the topic relate to the learners' lives, e.g. relate topics to the real world and possible career choices.
(3) Write key words in a larger font and use pictures or drawings to explain concepts. Underlining key words or phrases helps the learner to notice the language.
(4) Learners acquire and retain language through discovering the meaning themselves from within the text or through ordering a set of sentences to describe a process.
(5) Repeat the key language during the lesson in different exercises, e.g. use of key words, terms and phrases related to that topic, and the key words in assessment tasks such as 'Hence'.

(6) To help with developing thinking skills and using the language, start by asking simple questions using *what*, *when*, *where* and *which* followed by more challenging ones using *how* and *why*. This works well in group and pair work.
(7) Have a glossary and word bank available. Give the learners specific words and phrases to build the precise and more complex sentences they will need to be familiar with.
(8) Provide scaffolding – i.e. using activities where learners need to add correct answers from a choice to make the sentences correct. Follow this with practice of the same language used in sentences that have less scaffolding or have no scaffolding.
(9) Try to ensure pronunciation and word stress is correct. You can use delayed feedback. This means making notes during a group discussion time when the language may be difficult, and feeding this back to the whole group later in the lesson, just before the lesson review.

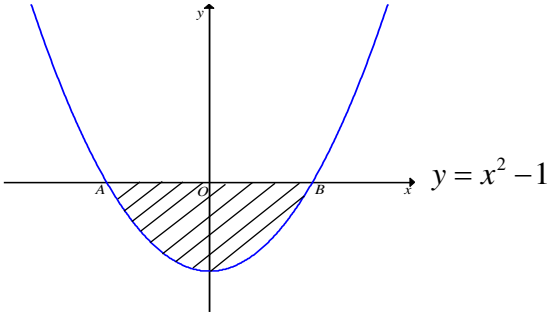
3.3.3 Some examples of exercises to integrate language learning within lessons

Exercise 1: Solution Ordering

An excellent starter or review activity could be to order the steps of a solution. The discussion that arises from such activities is very useful. Key words, phrases, terminology and notation can be explored. This exercise has the advantage of learners not needing to write anything down. They can be provided with the steps of the solution typed or written on strips of paper that they physically put in to the correct order.

You are given a question and the unordered steps of a solution.
Cut out the solution steps and place them in the correct order.

Question:
The graph of $y = x^2 - 1$ cuts the x-axis at A and B.
Find the shaded area.



Unordered solution steps:

$= 2\left[\frac{1}{3} - 1 - 0\right]$	$= 2 \times -\frac{2}{3}$	$x = \pm 1$	$2\int_0^1 (x^2 - 1)\,dx$	$= -\frac{4}{3}$
$= 2\left[\frac{x^3}{3} - x\right]_0^1$	Therefore shaded region $= \left\|-\frac{4}{3}\right\| = \frac{4}{3}$	$x^2 - 1 = 0$		

This type of exercise makes an excellent starter or review activity for a lesson. Some excellent discussion points can arise. For example:

- *Why have the limits 0 and 1 been used?*
- *Would an answer of 1.3 be acceptable?*
- *How does the negative value relate to the graph?*

- *As you can work this out on your calculator, would it be acceptable to just write the answer down?*

This discussion should help you assess your learners understanding of what they have learned as well as checking that they are all clear on what is expected to be given in a full solution.

Exercise 2: Using tables or flow diagrams

Again, this type of question makes a good starter. It can also be used as the initial question in a mixed exercise. The advantage of using this structure is that it reduces the number of words that learners need to interpret and allows them to concentrate entirely on the language of the mathematics. Two examples have been given to show how this could be used.

Example 1 shows how you could use a table to assess recall of facts. This simple activity links the terminology directly with the mathematical symbols and formulae. You could complete more of the table as a starting point if you wanted to provide more scaffolding. The table could be adapted to use specific values of *a*, *d* and *r* if you wanted to include it as part of an exercise.

Example 1

	AP	GP
first term	a	a
common difference		
common ratio		
*n*th term		
sum to *n* terms		
sum to infinity		

Put the information given below correctly into the table.

Some parts of the table will be empty.

d $\quad u_n = a + (n-1)d \quad u_n = ar^{n-1} \quad r$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_n = \frac{n}{2}(a + l)$$

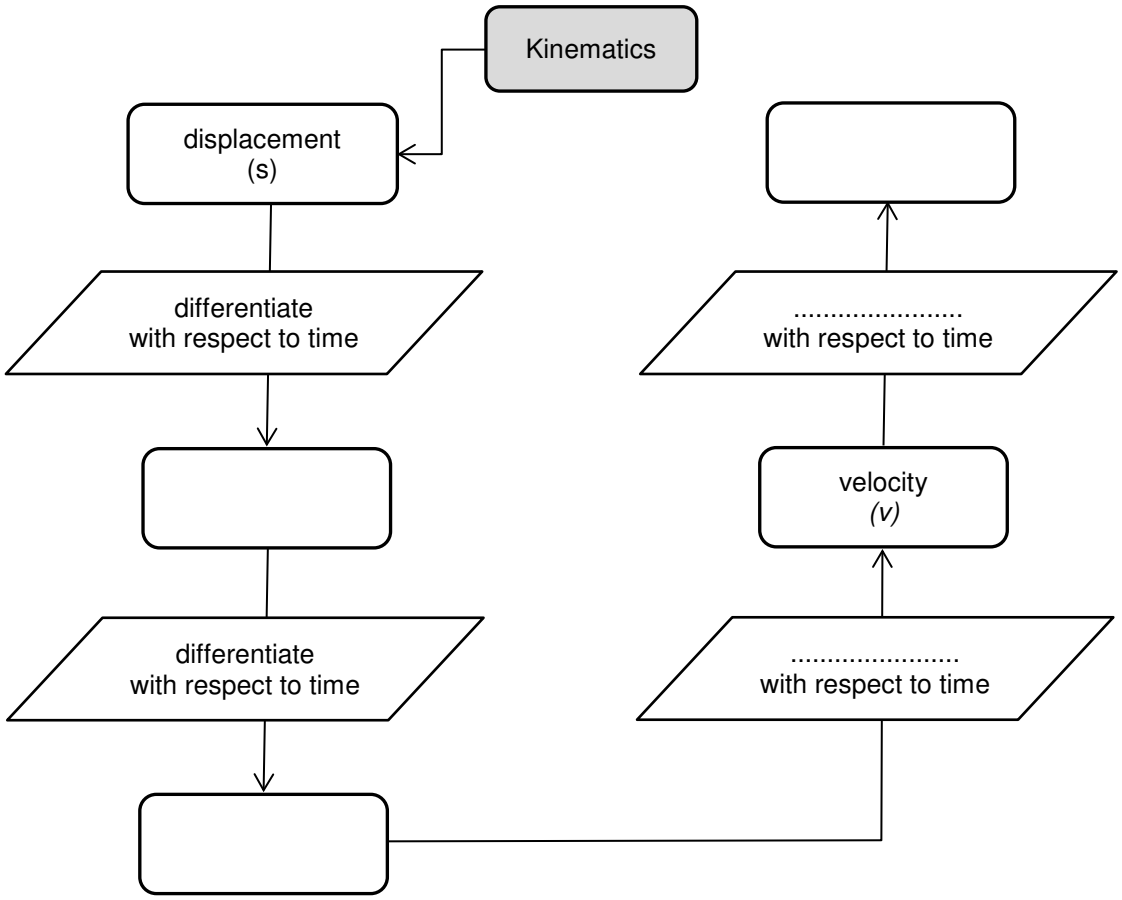
$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r}, \quad |r| < 1$$

Example 2 illustrates how you could use a flow diagram to show how different quantities are related. This would be a good starter or review activity. The mathematical notation for derivatives and integrals could be included as well if you wanted.

Example 2

Complete this diagram.



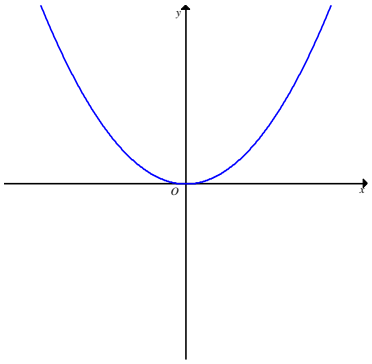
You could replace the general statements in each box with specific functions. Diagrams such as this neatly show the relationships between quantities without needing to interpret a large amount of language. The focus is the key words and terms.

Exercise 3: Matching exercises

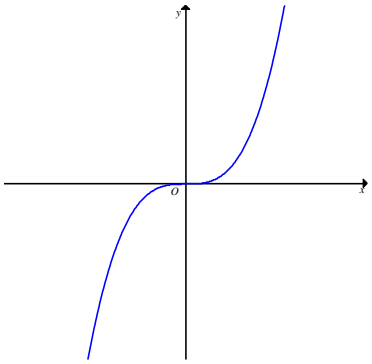
Using diagrams to match graphs to descriptions minimises the use of language 'output' so that learners can focus on interpreting the question.

The diagram shows 3 graphs.

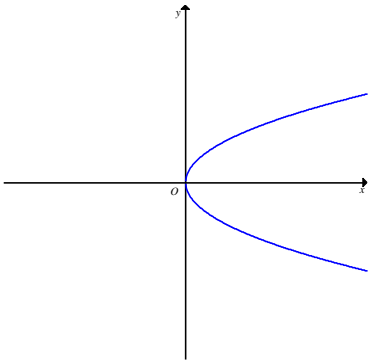
A



B



C



(a) Write down the letter of the graph(s) that represents a function that has an inverse.

(b) Write down the letter of the graph(s) that represents a function that does not have an inverse.

(c) Write down the letter of the graph(s) that does not represent a function.

Each part has the command words 'write down'. This is not a complex instruction and the learner is allowed to concentrate on interpreting what they have to write down. In this case it is demonstrating understanding that they know what the definition of a function is by simply matching the diagrams with the descriptions given.

Exercise 4: Assessing framed answers

In this type of exercise, learners are given the question and some sample answers. Learners assess the value of each answer they are given. This actively focuses their attention on the structure and level of detail that might be needed for a solution without actually constructing the answer themselves. Again, this allows them to concentrate on the language in the question.
In the type of question given in this example, learners need to 'show that' a stated result is true. This requires structured evidence, which is often missing from learners' attempts.

You are given a question and four answers written by different students.
Award each answer a mark from 0 to 3 and describe why you awarded that mark.

Question:

In a cubic polynomial, p(x), the coefficient of x^3 is 2.

The roots of the equation p(x) = 0 are $x = -5$, $x = 0.5$ and $x = 2$.

Show that p(x) = $2x^3 + 5x^2 - 23x + 10$. [3]

Answers to mark:

Answer 1:

$p(-5) = 2(-5)^3 + 5(-5)^2 - 23(-5) + 10$
$0 = -250 + 125 + 115 + 10$

$p(0.5) = 2(0.5)^3 + 5(0.5)^2 - 23(0.5) + 10$
$0 = 0.25 + 1.25 - 11.5 + 10$

$p(2) = 2(2)^3 + 5(2)^2 - 23(2) + 10$
$0 = 16 + 20 - 46 + 10$

Answer 2:

Roots are $x = -5$, $x = 0.5$ and $x = 2$
Factors are $x - 5$, $2x + 1$ and $x + 2$

$p(x) = (x - 5)(2x + 1)(x + 2)$
$= 2x^3 + 5x^2 - 23x + 10$

Answer 3:

Factors are $x + 5$, $2x - 1$ and $x - 2$

$p(x) = (x + 5)(2x - 1)(x - 2)$
$= (x + 5)(2x^2 - 5x + 2)$
$= 2x^3 - 5x^2 + 2x + 10x^2 - 25x + 10$
$= 2x^3 + 5x^2 - 23x + 10$

Answer 4:

$p(x) = (x + 5)(x - 0.5)(x - 2)$
$= 2(x + 5)(x - 0.5)(x - 2)$
$= 2x^3 + 5x^2 - 23x + 10.$

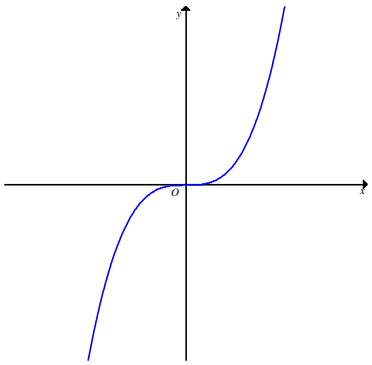
The linguistic demand is in interpreting the question. This exercise requires more complex reasoning skills than Exercise 1 however, as the marks given should be justified.
Working in pairs or small groups, learners need to give each answer a mark from 0 to 3. Learners really need to think about what is required for the full 3 marks. The results can be shared and compared and discussion of what a good solution looks like should arise. The appropriate structure for the best solution can be modelled at the end and agreed upon. The need to check answers should also be part of the discussion.

Exercise 5: Explain

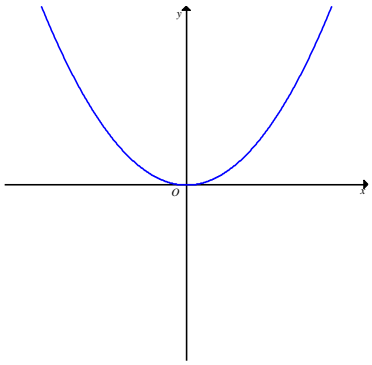
Learners are often required to provide a justified explanation for a stated result. In these cases, explanations are sometimes incomplete. Sometimes learners language is not correct or insufficiently accurate for them to be awarded marks.

The diagram shows three graphs.

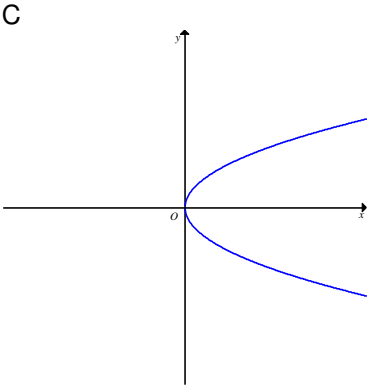
A



B



C



(a) Explain why graph A represents a function that has an inverse.

(b) Explain why graph B represents a function that does not have an inverse.

(c) Explain why graph C does not represent a function.

Questions of this type require a much higher level of linguistic ability. The learner must give a justified explanation to support the statement made. Sometimes learners contradict themselves when giving such explanations. Sometimes learners offer too little information to be convincing. When using past paper questions of this type, the mark scheme for the question can be considered by the group once they have completed their solutions. This allows learners to check that their solutions are appropriate and sufficient.

3.4 Problem Solving

The Cambridge IGCSE Additional Mathematics 0606 course has two Assessment Objectives (AO). One has a focus on demonstrating knowledge and understanding of mathematical techniques (AO1), the other has a focus applying mathematical techniques (AO2):

AO2 Apply mathematical techniques

Candidates should be able to:

- *recognise the appropriate mathematical procedure for a given situation*
- *formulate problems into mathematical terms and select and apply appropriate techniques.*

This AO requires the skill of 'problem solving'. Questions targeting AO2 make up 50% of the total marks for the assessment. Therefore, it is essential that your learners develop good problem-solving skills and are able to apply a combination of the methods and techniques they have learned to answer questions successfully.

3.4.1 Showing their working

Make sure your learners are aware of the following rubrics, which appear on the front of the examination papers:

- *You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.*
- *Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.*

These rubrics are particularly important when problem-solving or answering questions that have multi-step solutions; you should discuss these rubrics at appropriate times throughout the course so that it becomes natural to the learner to provide a fully supported answer, when appropriate.

Learners are expected to have an electronic calculator and should know how to use it efficiently. Encourage the use of calculators mainly to check answers, and not to be used as a substitute for doing procedures manually. (There are of course exceptions when calculations are better done on the calculator, such as when solving trigonometric questions.) This is important because for some questions, they will not be awarded marks if they have not shown all the necessary method steps.

Learners are advised that non-exact answers should be given to 3 significant figures. The exception to this rule is angles measured in degrees, which should be given to 1 decimal place. This means that any working values show in their working must be stated to **more than** 3 significant figures, or **more than** 1 decimal place in the case of angles in degrees, to avoid a premature approximation error. Understanding the significance of error and accuracy in answers is something that is essential to any learner who intends to continue their mathematical career at higher levels.

3.4.2 Problem-solving strategies

Once learners are aware of the need to show their working and only use a calculator for support (as appropriate), you could suggest a strategy for them to adopt when approaching problem-solving questions. Some learners are excellent problem-solvers and will intuitively work through the steps of a problem and successfully find the solution. Other learners may need support in this area. You may choose to encourage your good problem-solvers to share their strategic approaches. Teaching others is an excellent learning process.

Some learners might benefit from a structured approach such as:

Given – What information is given in the question?
Asked for – What are you trying to find out?
Unknown – What do you not know that you need to know?
Strategy – What maths do you need to use?
Solve – Now go ahead and solve the problem.

This framework can be applied to any problem and should enable learners to think critically and discover what is needed for themselves.

3.4.3 Investigative approaches

Learners are often surprised that there can be more than one successful solution to a problem and that all solutions can be equally valid. A good way to demonstrate this, is to give investigative tasks to groups of learners, and ask them to share their results; they might discover that there are several different, but correct,

solutions. Learners might also deepen their understanding of what they have been studying when they are given the chance to present their solutions to the class.

Investigative tasks take longer than most other teaching methods. It is important to allow enough time for any task to be successfully completed. Timing will be quite important for this type of lesson. When setting up a task of this type, providing some questions that are 'scaffolded', or have single answers, early in the task should help all learners to engage with it. Try also to include some questions that may have a variety of answers, rather than just one answer. These are called 'open' questions. When appropriate, ask for answers or decisions to be justified as learners find this especially challenging and need to practice this skill. You may also choose to provide different investigative tasks to differentiate across the ability levels of your learners – so the better learners in the group may have a more challenging task than the majority, for example.

Even when they cannot immediately see the solution to the problem, learners should be encouraged to try to take what seems to be a sensible first step and then see where it leads. Try to give positive feedback when possible. Learners need to learn from their mistakes and value them as learning opportunities, and to not consider themselves to have failed. Encourage learners to discuss what went wrong, if errors have occurred, and what could be done to change that.

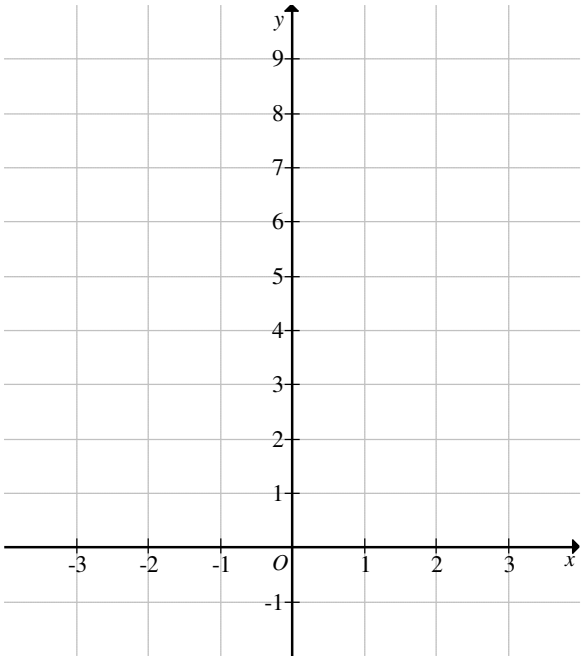
This type of work is best done in pairs or small groups. Some groups may be doing one task whilst other groups are doing another. The groups undertaking the same tasks could pool their results before presenting to the class as a whole. This gives another opportunity for information to be shared and considered, learner to learner and should deepen their understanding.

3.4.4 Sample Investigation

On the next few pages there is an example investigation that could be given to learners. Teacher notes to support the investigation follow in the next section.

Task 1

1. (a) On the diagram below, draw the graphs of $y = |2x + 1|$ and $y = 4$



(b) Use your graph to solve $|2x + 1| = 4$.

.....

(c) Write down the critical values for the inequality $|2x + 1| > 4$.

.....

(d) Use your graph and your answer to part (b) to solve $|2x + 1| > 4$.

.....

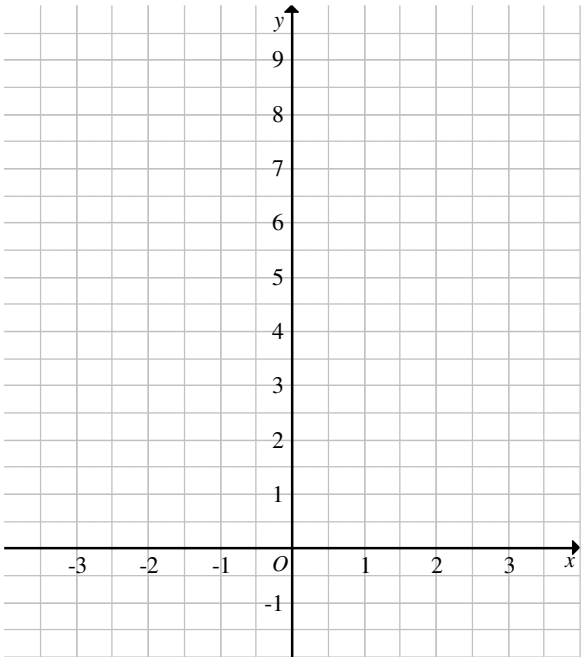
(e) Explain how you can check your answer to part (d).

.....

.....

2. Use a graphical method to solve $|5x + 2| \leq 2.5$

Complete the method below the graph by writing down each step of your method.
(You may use more than 2 steps).



Method

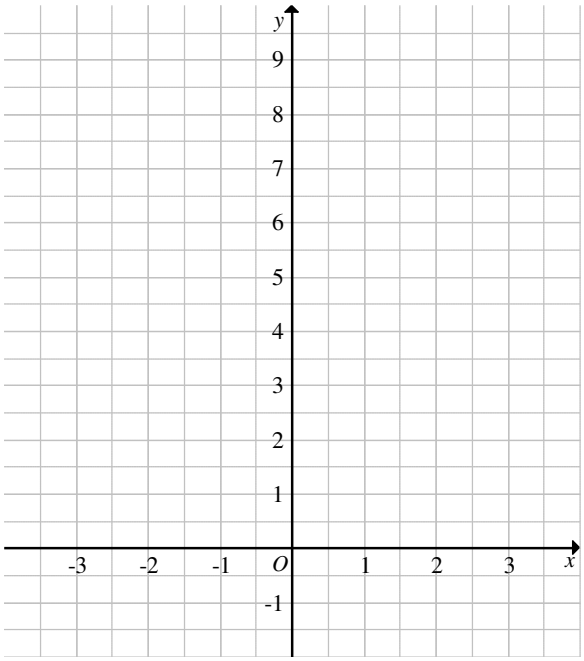
Step 1 Draw the graphs of .....

Step 2

Task 2:

1. Use a **graphical** method to solve $|5x + 2| \leq 2.5$

   Complete the method below the graph by writing down each step of your method.
   (You may use more than 3 steps).



Method

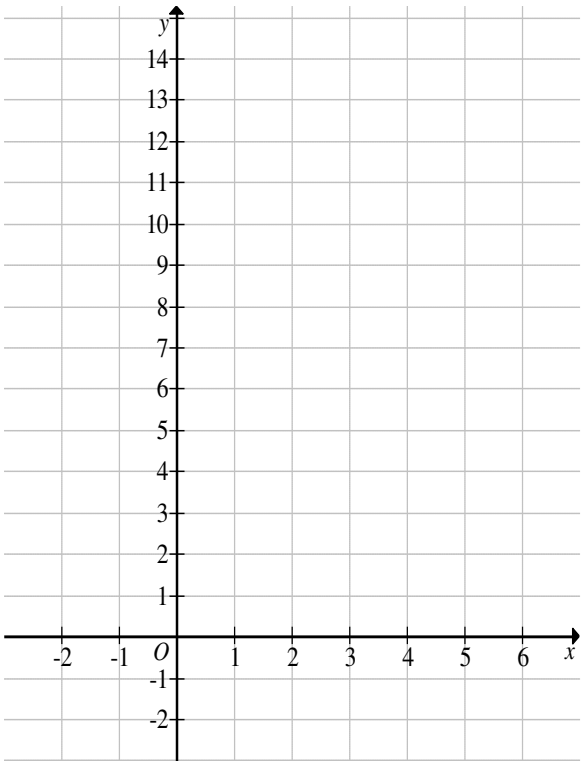
Step 1 Draw the graphs of $y = |5x + 2|$ and .....

Step 2 Use the graph to find the points .....

Step 3

2. (a) Use a **graphical** method to solve $|3x-7| \leq 4x+1$.

Complete the method below the graph by writing down each step of your method. (You may use more than 2 steps).



Method

Step 1

Step 2

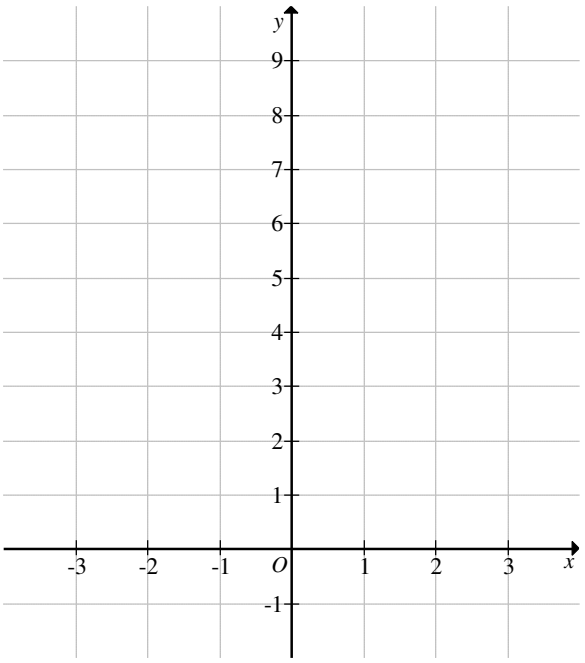
(b) Explain how you can tell from your graphs that $|3x-7| \leq 4x+1$ has only one critical value.

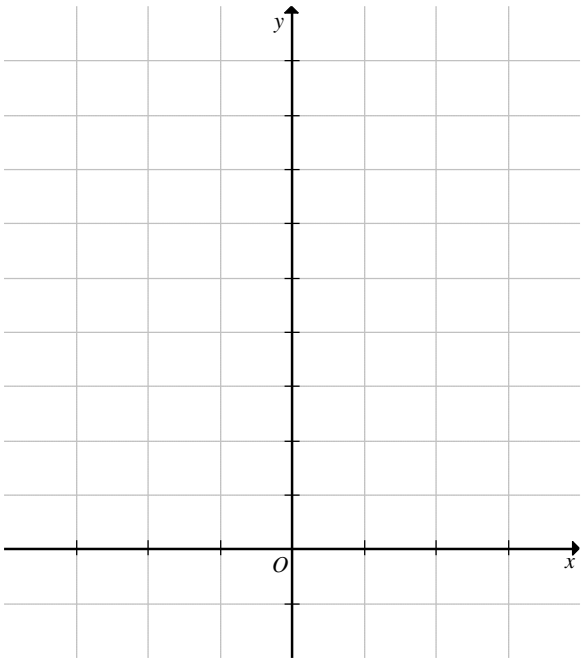
.....

.....

Task 3:

1. Find a **graphical** method to solve $|ax + b| > c$.
   Complete the method below the graph by writing down each step of your method.
   You may use the grids to help you.
   (You may use more than 3 steps).





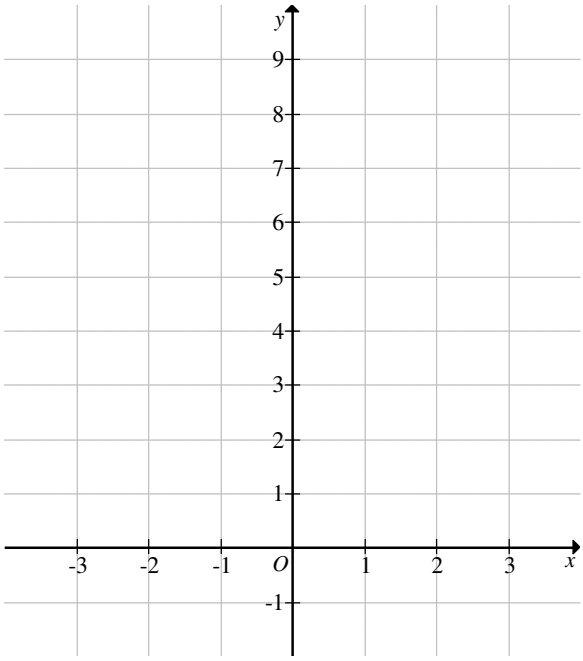
Method

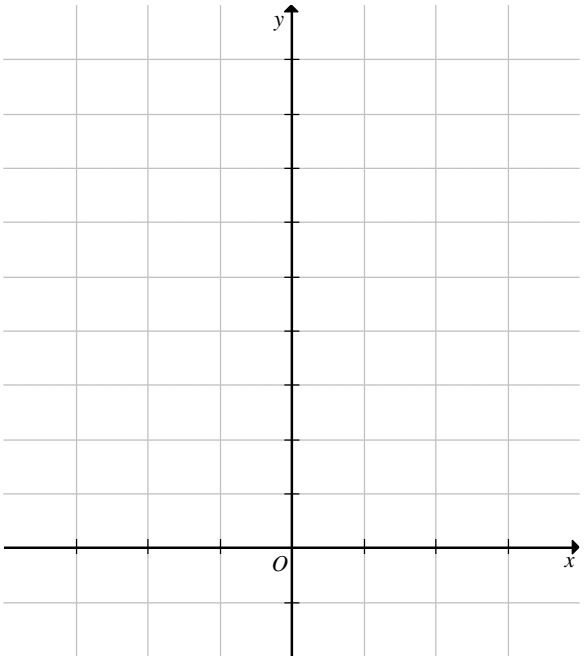
Step 1 Draw the graphs of .....

Step 2 Use the graph to find the points .....

Step 3

2. (a) Find a **graphical** method to solve $|ax+b| \leq cx+b$ when the value of a is greater than the value of c.
Complete the method below the graph by writing down each step of your method.
You may use the grids to help you.
(You may use more than 2 steps).





Method

Step 1

Step 2

(b) Explain why it is possible for $|ax+b| \leq cx+d$ to have no critical values, one critical value or two critical values or find a specific example of each case.

.....

.....

3.4.5 Sample investigation teacher notes

Syllabus subject content:
3 Equations, inequalities and graphs

- solve graphically or algebraically inequalities of the type $|ax+b|>c\,(c\geq 0)$, $|ax+b|\leq c\,(c>0)$, $|ax+b|\leq(cx+d)$.

Resources:
Investigation worksheets; graphing software.

Points to note:
The aim of the investigation is for learners to devise a **graphical** method of solving inequalities of this type.

Task 1 – Offers the most support and structure and should be accessible to all learners. The functions being worked with are specific cases.
Task 2 – Offers some support and structure and should be accessible to most learners. The functions being worked with are specific cases.
Task 3 – Offers little support and structure and is a much stronger test of problem-solving skills; it is more appropriate for the best learners in the class. The functions being worked with are general cases. Learners will need to generate their own specific cases to establish a method.

It will help learners' confidence if they are not aware that the tasks have been differentiated in this way.

The task is not assessing how well learners can draw graphs of modulus functions. Therefore, it is important that graphing software such as Geogebra or Desmos, or graphical calculators, are available for them to sketch the graphs they need. They should find intersections using the software but graph paper has been provided on the worksheet so that they can make a sketch and record their findings. This is also a good opportunity to highlight the skill of sketching a graph as oppose to drawing one.

Part of the information gathered by your learners doing tasks 2 and 3 should be the number of possible critical values there are when the inequality is of the form $|ax+b|\leq(cx+d)$, and how that changes the structure of the solution. This needs to be discussed at the end so that all learners have an understanding of why this is possible. It is important that learners are comfortable with, and can interpret, cases that may have no critical values or only 1 critical value.

Examples are:

no CVs $y=|3x+2|$, $y=x-1$
one CV $|3x-7|\leq 4x+1$, from task 2
two CVs $|3x-7|\leq x+1$.

Suggested open questions to ask learners:

- The method needed is 'graphical'. Where do you think is a good place to start?
- Can you solve a simpler problem instead? (You could lead them to, e.g. $|5x+2|=2.5$, but they should think of this option for themselves, with your guidance.)
- Why did you try that?
- What do you think comes next? Why?
- You have found some critical values. How can you use them?
- How could you check your answer?
- Does your answer make sense?
- Is your method clearly stated?
- What happens when you change the values of *a* and *c*?

Possible misconceptions:
Learners often think that x cannot be negative when dealing with modulus functions. It would be useful to emphasise that it is the value of y that is always positive and x can be positive or negative. Mentioning this whilst learners are working on the graphs is an excellent time to clear up this misconception.

3.4.6 Sample lesson plan

Below is a sample lesson plan that puts some of the language learning, problem-solving and investigation tasks together into a single lesson.

Lesson:		**School:**
Date:		**Teacher name:**
Class:	**Number present:**	**Number absent:**
Learning objectives to which this lesson is contributing	Solve graphically or algebraically inequalities of the type: $\|ax+b\| > c\ (c \geq 0)$ $\|ax+b\| \leq c\ (c > 0)$ $\|ax+b\| \leq (cx+d)$	
Lesson objectives	You will develop methods to be able to solve inequalities of the following type, using graphs: $\|2x+1\| > 5$, $\|6x-7\| \leq 8$, $\|3x+2\| \leq x+1$	
Vocabulary, terminology & phrases	Modulus, greater than, less than, greater than or equal to, less than or equal to, critical values, set of values, linear, intercepts, roots	
Previous learning	Learners should already be able to: • sketch the graphs of, for example, $y = \|3x-1\|$, $y = 4x+3$, $y = 2$ • solve equations such as $\|3x-1\| = 2$, $\|3x-1\| = 4x+3$ • find the critical values to solve a quadratic inequality.	

Plan

Planned timings	Planned activities	Resources
Beginning (5 min)	Starter activity: Present the class with 10 Multiple choice questions using PowerPoint. For each question, include a graph of a linear function, or the modulus of a linear function, along with 3 possible equations to represent the graph (A, B, C). Learners write down which they think is correct (or hands up).	PowerPoint; pencils/pens; paper.

Plan		
Planned timings	**Planned activities**	**Resources**
Middle (50 min)	Set an investigation to solve modulus inequalities. Arrange learners into small groups of 3 or 4 and assign them a task (1–3) depending on their abilities. Task 1 – devise a method for solving $\|2x+1\| > 4$ $\|5x+2\| \leq 2.5$ Task 2 – devise a method for solving $\|5x+2\| \leq 2.5$ $\|3x-7\| \leq 4x+1$ Task 3 – find a method for solving $\|ax+b\| > c$ $\|ax+b\| \leq cx+b \quad a > c$ Ask like groups to collect together their result, i.e., Task 1 groups to pool their information; Task 2 groups to pool their information; Task 3 groups to pool their information. Ask leaners from each 'task group' to present the results to the rest of the class in the order 1, 2, 3. Summarise the results of the investigation. Discuss any misconceptions.	Worksheets for Tasks 1 –3; pencils/pens; paper; graphing software.
End (5 min)	Ask learners to write down what they think are the key words or phrases from the lesson. Ask them for at least 3. Give them 2 minutes to do this and then invite learners to share their results. Examples might include: intercepts/roots, y always positive, critical values, less than, greater than, less than or equal to, greater than or equal to, check answer, linear, modulus, set of values. Write these on the board or a large piece of paper for reference at the start of the next lesson.	Pencils/pens and paper.

Additional information		
Differentiation: How do you plan to give more support? How do you plan to challenge more able learners?	**Assessment: How do you plan to check learning?**	**Health and safety check: ICT links**
Differentiation by task: There are 3 tasks to provide suitable challenge for learners of all abilities.	Observation, questioning and answering throughout activity: walking around whilst tasks are ongoing and asking directive questions; discussion.	All equipment inspections are up to date and booked suitable computer room.

Reflection and evaluation	
Reflection *Were the lesson objectives realistic?* *What did the learners learn today?* *What was the learning atmosphere like?* *Did my planned differentiation work well?* *Were the timings correct?* *What changes did I make from my plan and why?*	**Use the space below to reflect on your lesson. Answer the most relevant questions from the column on the left about your lesson.**

Summary evaluation

Write down two things that went really well. (Consider both teaching and learning.)

1.

2.

Write down two things that would have improved the lesson. (Consider both teaching and learning.)

1.

2.

What have I learned from this lesson about the class or individuals that will improve my next lesson?

3.5 Risk assessment

It is important to assess any risk involved in, for example, using a computer room or setting up laptops by learners. It is important to make sure that there is sufficient room for learners to be safe; that the lighting is adequate; and that the temperature of the room is not too high.

Any electrical equipment should be inspected regularly and the use of extension leads for laptops, for example, should be kept to a minimum. Any tasks should not be for too long a period of time to avoid headaches and eye strain.

Centre policy should be followed. Examples of good practice are available on the internet. These factors should be taken into account when deciding on an activity.

4 Preparing learners for final assessment

You will find past papers and mark schemes on the School Support Hub (www.cambridgeinternational.org/support). These can be used by learners for exam practise and/or for formative assessment throughout the course or at the end of a topic. You will also find the Principal Examiner Reports for Teachers (PERT), which is produced after each examination series. The PERT indicates the strengths and weaknesses of candidate performance across the whole cohort and can be used to help you identify common areas of misconception, misunderstanding and weakness in order to improve your teaching.

4.1 Study habits

By the start of the IGCSE course, learners will probably have explored preferred methods for studying and revising. However, not all of these methods are necessarily effective for all learners.

Much research has been published on this subject, suggesting that some of the following methods are not effective (though of course it is depends on the individual):

- generous use of highlighters
- reading and re-reading notes
- working exhaustively and alone
- re-writing existing notes to create a more attractive set of notes.

Dedicated learners will often revise intensely for long periods and convince themselves that they have prepared thoroughly. Sadly, they might have been largely wasting their time, especially if they are aiming to develop a deep and lasting understanding of the topic, in addition to just passing the examination.

Here are some methods that are proven to work for **most** learners:

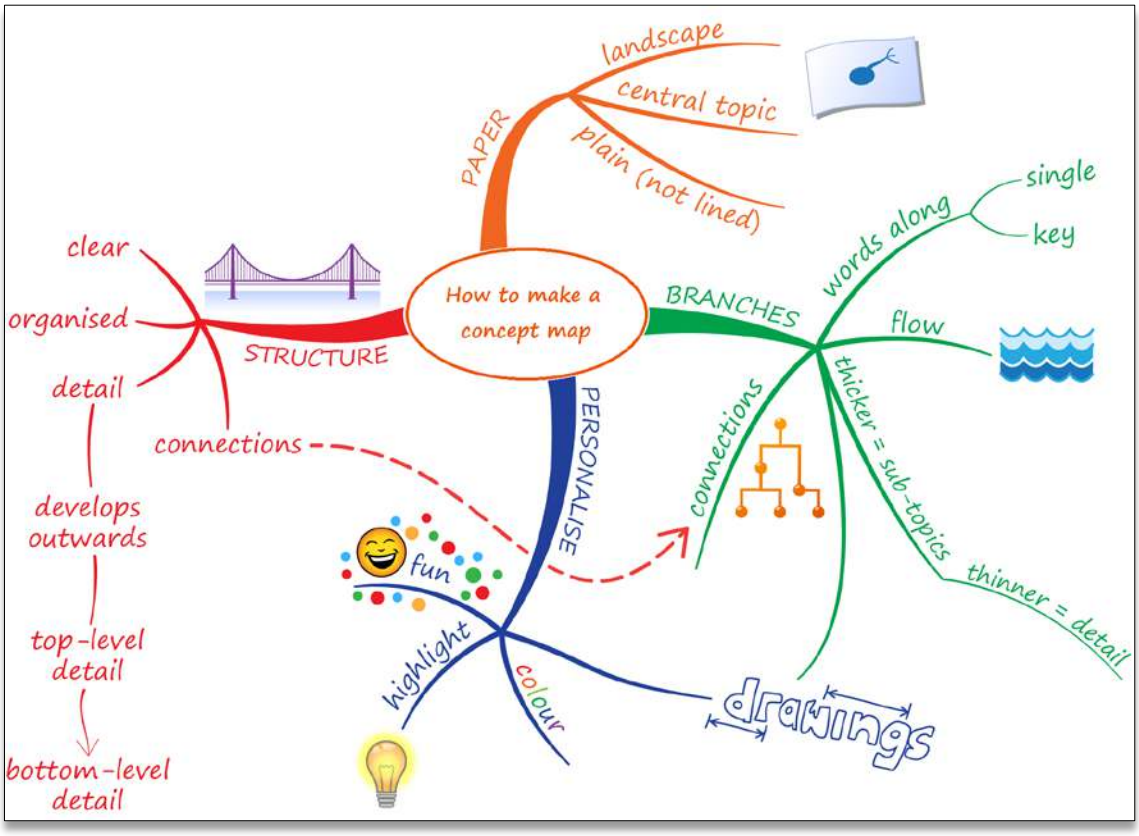
- Distributive practice: that is, spreading out study over time. This method is believed to aid true understanding of the topics.
- Studying in short bursts, followed by testing themselves regularly over several weeks.
- At the end of a revision session, writing down what they can remember.
- Creating a revision timetable for the mock and final exams. This will ensure that they study different subjects in small amounts but often.
- Answering lots of practice questions/past papers.
- Connecting ideas together by the use of concept maps.
- Using revision guides rather than the subject textbook.

Encourage your learners to consider and develop what works best for them. There is a Learner Revision Guide (www.cambridgeinternational.org/images/351658-learner-revision-guide-.pdf) available on our public website that provides some general guidance.

4.2 Deep subject understanding

When learners start to make connections between topics, the study of the subject becomes more enjoyable and they gain a deeper understanding of what they are doing.

Concept maps (see diagram below) can be drawn and connections made between sub-topics in a unit, between units in a syllabus, and indeed between related subjects.



4.3 Technology in and out of the classroom

There are a great range of technological tools available for use inside and outside of the classroom. It is important for learners to experience their learning in a variety of different ways, not least to maintain interest and motivation. Below are just some of the possibilities.

4.3.1 Graphing Software

Learners often have better recall when they can visualise a solution. The appropriate use of graphing software to investigate, for example, transforming trigonometric graphs, will make the topic much more meaningful and memorable than simply being given a list of rules to learn. Geogebra (www.geogebra.org) and Desmos (www.desmos.com) are two such software packages that could be used to do this.

4.3.2 Mobile apps

Mobile apps for education have to be carefully selected to make sure they provide appropriate and meaningful learning outcomes. But if chosen well, they can provide another mode of learning or revision for the learner. There are numerous options, from games and quizzes to videos and animations.

'Socrative' is an excellent app for formative assessment and learners usually enjoy using it. You can create online multiple-choice style quizzes that give instant feedback to you, the teacher, so that you can quickly identify problem areas. Correction and explanation can then be dealt with immediately.

4.3.3 Podcasts

These audio teaching aids are a handy alternative tool and can be especially useful when learners are travelling to and from school or do not want to disturb others. Listening to the same podcasts over and over again can be especially useful for the second language learner. You can create your own podcasts online for free at 'Podbean', for example. Creating your own podcasts allows you to choose the emphasis you want and use the language you have been specifically using with your learners.

4.3.4 Video

Video is not just something that learners sit down and watch in order to add variation to a lesson. Videos can be stopped periodically and questions asked in the traditional way, or they can even be edited and teacher questions inserted/embedded within the video itself. This makes the process much more active, which increases learning potential. You could make your own or have a look at the many examples on YouTube and the Khan Academy, for example.

4.4 Providing feedback on learner work

It is necessary to provide meaningful feedback to your learners in order for them to improve the quality of their written answers and understanding of a subject. The learner may find the subject itself challenging and/or may not have the skills in English to deal fully with the question. Either way, meaningful feedback and reflection time are the answer. Providing feedback can be done in several ways, such as:

- Provide feedback orally in class so that learners write down the extra information they would need to get full marks.
- Allow learners individually to find the other pieces of information to get full marks.
- Provide a situation where the learners are actively engaged in reviewing the questions in pairs or groups with your guidance. In this way they can collectively understand what information and language would have gained full marks. You can point out command words used in the question and encourage the learners to develop an understanding of what each word means.

The last suggestion however, takes time and it could be set as a group exercise to be started in learners' own study time.

Technology is available to allow you to easily record units of your own teaching in short, manageable portions. These can be made available to learners who can watch them as a homework assignment. This saves time and allows you to concentrate on other aspects of learning and allows more time for formal assessment. The fact that these videos can be watched again and again is especially useful to the second language learner.

4.5 The mock examination

The mock examination is an important benchmark for teachers and learners. It serves several purposes:

- It is an opportunity for the learners to be tested on the complete course material* under proper exam conditions.
- It encourages learners to start revising for the exams earlier (otherwise they might put off revision until only a month or less before for the real exam).
- It allows learners to become more familiar with the process of being examined, so that on the day of the actual examination they might feel less stressed and more confident – this is especially important for learners who are apprehensive or nervous about taking exams.
- It provides an opportunity to spread the revision load of the subject over several months.

** Even if learners have not completed the course by the time of the mock exams, an exam should be created that allows the learner to get the mock exam experience on the majority of the syllabus content.*

After reviewing the results of the mock exam, learners might gain an insight into the following:

- how successful their revision techniques had been
- which topics and sub-topics need more revision
- if they had enough time to complete the exam and check through their answers
- if they were able to perform properly under pressure
- whether there are questions in which they would have gained the mark(s) had their English been clearer
- if they lost marks because of not being clear and/or not using the correct terminology.

If learners treat the mock exams as if they were the real examinations, evidence indicates that they might perform even better in their real final assessment. Some learners might take considerable persuasion to take the mock exams seriously enough to revise properly. They need to be encouraged to appreciate that the process is a positive and supportive one, and one in which very useful feedback will be provided.

4.6 Use of past papers

Working through large quantities of past papers is a major factor in the success of a learner in the final assessment. Past papers allow learners to practice the *type* of questions that might come up in their final exams. However, it is important to keep in mind that mark schemes contain several alternative acceptable answers. This provides an opportunity for learners to suggest the best possible answer. Useful discussion might arise about the syllabus area on which the question is based.

Be aware of any data provided in the syllabus and, importantly, any changes to this data that might affect how learners understand any practice questions used from past papers.

The Principal Examiners Reports for Teachers (PERTs) are very helpful to use in conjunction with mark schemes. They indicate specific areas of the syllabus where learners performed well or need to improve. Some of these areas of weakness are mentioned year after year which might help you decide on the length of time you should spend on certain areas or how you arrange the order of your teaching.

Learners need to be reminded that:

- In multiple-mark questions they should,
  - underline key words and what exactly the question requires them to do
  - take note of the number of marks available as this will indicate the extent of the answer required; answers often appear as if the learner has not noticed that there are 5 marks available for example
  - show every step of a calculation (even if doing so appears unnecessary to able learners who arrive at the final answer easily); learners often do not realise that 'method marks' can be awarded even when their final answer is incorrect, or that method marks are sometimes required in order for the final accuracy mark to be awarded
  - re-read the question after they think they have completed their solution as there is often more than one requirement in a question; learners sometimes omit parts of answers because they have simply forgotten what they initially read.

- The front page of the examination paper contains very important information about the use of calculators and the accuracy to which non-exact answers should be stated.
  - Learners need to understand that in a question that has no context, an exact answer in the form of $\frac{4}{3}$ for instance, is perfectly acceptable; decimalisation of such fractions is not necessary as the answer can be given exactly. Other exact forms may involve surds, π, e or logarithms.

- Non-exact answers are required to be given to 3 significant figures (unless the question states otherwise, and apart from angles given in degrees). Therefore, any working values used must be more accurate than 3 significant figures. When learners do not do this, they introduce a premature approximation error and their final answer becomes inaccurate.

- They should present their work in a clear, logical, step-by-step way. If a learner wants to re-answer a question or change a solution, it is better to use a fresh sheet of paper or blank page, rather than squashing solutions into any working space remaining. Poorly presented work can result in learners misreading their own writing and result in unnecessary errors occurring. In the examination, it is useful if learners indicate where they have continued their solution if this is the case.

- When a question indicates that a calculator must **not** be used, learners must ensure that they show **all** working steps and arithmetic, in order to provide sufficient evidence that they have followed this instruction; without the supporting working marks will not be awarded.

- When a question indicates that solutions by accurate drawing are **not** acceptable, learners should know that it is still fine for them to make sketches to help in their working. The instruction in this case is to avoid answers that have been found using scale or accurate drawings when an *algebraic* solution is required. Sketches and diagrams are good ways to visualise the mathematics and should be encouraged.

- Take care to be clear when answering questions involving comparing two or more pieces of data: it is often unclear which piece of data the learner is discussing and the comparative language is not used properly.

4.7 Command words

It is important that learners understand the vocabulary of the assessment objectives and the 'command words' of exam questions that indicate the approach they should take to an answer. For example, 'calculate', 'describe' or 'determine'.

The 'command words' are listed in the syllabus. Think about the skills level required to carry out each of the commands listed. For example, 'write down' or 'verify' indicate less complex tasks than 'show that' or 'explain' since the latter cases require full and structured evidence to be presented. Another common supporting word is 'Hence...'; this is not listed in the command words but it is important that learners understand the significance of this word.

You can use the Example Candidate Response (ECR) booklet to help explain the meaning of the command words to learners. The ECR booklet contains candidate responses that have been marked as 'high', 'middle' or 'low' grades and is produced after the first examination has been sat. Showing learners good sample responses to questions using different command words can help them see how increasing levels of skill relate to the marks available. Asking questions in ascending order of skill whilst teaching a topic will encourage learners to ask themselves similar questions when they are learning alone. When they become practiced at this, they will begin to see patterns emerging where the same processes and logic can be applied to similar scenarios even if they have not been covered in class.

5 Resources and support

5.1 Finding and evaluating resources

There is no shortage of resources to aid the teaching of mathematics. They can be found in text books and on the internet. The problem is not so much finding resources, but evaluating whether they will suit your situation and are effective. The quality of resources varies widely from 'homemade' ones that are uploaded to the internet, to professionally produced ones.

When possible, the easiest way to find reliable resources is to get them from a colleague who has already used them and can tell you how good they are.

You can also find resources on the Cambridge Support Hub (see section 5.2 below).

Resources from the internet and from books need to be scrutinised to see if they are of use. Some websites that may be useful are:

www.geogebra.org
Solve equations, graph functions, create constructions, analyse data, explore maths in 3D.

www.desmos.com
Graph functions, plot data, evaluate equations, explore transformations and much more.

www.stem.org.uk
Subject-specific, quality-assured resources designed to inspire young people.

www.nrich.maths.org
Free enrichment material: games, articles and problems.

www.risps.co.uk
A collection of 40 open-ended investigative activities, rich starting points for A level mathematics, many of which are suitable for differentiation for this syllabus.

5.2 School Support Hub

From the School Support Hub (www.cambridgeinternational.org/support), you will be able to access the syllabus and copies of past papers together with their mark schemes, Principal Examiner Reports and grade thresholds. You will also be able to download a scheme of work – this is just one example of a possible route through the syllabus with some suggested teaching activities. There is also a list of resources including ECRs, and a link to the 'Discussion Forum' where teachers can post comments and questions. It is worth looking at this from time to time and following interesting threads even if you do not post any comments of your own.

5.3 Training

The School Support Hub and Cambridge Events calendar (www.cambridgeinternational.org/events) on our public website has a list of upcoming training events. You can also register for these courses on the site. These include:

- online self-study and tutor-led courses; the tutor-led courses are highly recommended to help you improve your teaching skills – these are available at Introductory level for new teachers and Extension level for those who have been teaching IGCSE / O Level Additional Mathematics for around a year

- face-to-face courses; held at venues all over the world at different times throughout the year; these enable you to meet up with other teachers, and also to interact directly with a trainer from Cambridge.

In addition, Cambridge runs professional development courses for teachers who want to develop their thinking and practice. These include a range of Cambridge International Certificate and Diploma level programmes in:

- Teaching and Learning
- Educational Leadership
- Teaching Bilingual Learners
- Teaching with Digital Technologies

You can find information about these at www.cambridgeinternational.org/qualifications/teacher.

Appendix: Sample lesson plan template

Lesson:	**School:**	
Date:	**Teacher name:**	
Class:	**Number present:**	**Number absent:**
Learning objectives to which this lesson is contributing		
Lesson objectives		
Vocabulary, terminology and phrases		
Previous learning		
Plan		
Planned timings	**Planned activities**	**Resources**
Beginning		
Middle		
End		
Additional information		
Differentiation: How do you plan to give more support? How do you plan to challenge the more able learners?	**Assessment: How do you plan to check learning?**	**Health and safety check; ICT links**

Reflection and evaluation	
Reflection Were the lesson objectives realistic? What did the learners learn today? What was the learning atmosphere like? Did my planned differentiation work well? Did I stick to timings? What changes did I make from my plan and why?	**Use the space below to reflect on your lesson. Answer the most relevant questions from the box on the left about your lesson.**

Summary evaluation
What two things went really well? (Consider both teaching and learning.) 1. 2. **What two things would have improved the lesson? (Consider both teaching and learning.)** 1. 2. **What have I learned from this lesson about the class or individuals that will inform my next lesson?**

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